# EFFECT OF NONUNIFORM SENSITIVITY DISTRIBUTION ALONG A WIRE THERMOANEMOMETER PROBE ON TURBULENCE MEASUREMENT

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The effect is determined of nonuniform distribution of temperatures along a wire probe on the averaged turbulent velocity pulsation along it.

When turbulent pulsations of velocity are measured a measurement error arises due to averaging over the length of the probe.

This error was previously analyzed by assuming uniform sensitivity distribution along the probe [1]. In actual fact, however, the sensitivity is distributed along the probe in a nonuniform manner due to temperature distribution along its length. The end of the probe becomes cooler due to massive holders and this results in a shortening of the effective length. The latter modifies the averaging effect over the probe on the thermoanemometer readings. The estimation of the effect of averaging of the velocity pulsation along the probe with nonuniform temperature distribution is the subject of this article.

It is assumed that by employing one of the well-known methods the heat drift has been eliminated or can be ignored.

In this case the temperature distribution along the probe thread is governed by the equation

$$\frac{d^2\theta}{dx^2} - a^2\theta = -\frac{4Q}{\lambda_2 \pi d^2}, \tag{1}$$

where

$$a^2 = 4\frac{\mathrm{Nu}}{d^2}\frac{\lambda_1}{\lambda_2};\ \mathrm{Nu} = \frac{\alpha d}{\lambda_1};\ Q = I^2\rho;\ \mathrm{Re} = \frac{Vd}{\nu}.$$

The Nusselt number can be found from the relation [2]

$$\mathrm{Nu} = 0.42\mathrm{Pr}^{0.2} + 0.57\mathrm{Pr}^{0.33}\mathrm{Re}^{0.5}. \tag{2}$$

The temperature and velocity are each represented by a sum of two components, the averaged and the pulsation one:

$$\theta = T + t,\ V = U + u. \tag{3}$$

The averaged temperature T is constant in time but variable along the wire. The averaged velocity U is constant both in time and space. The variable components t and u are random functions of time and of the coordinate x. Moreover, it is assumed that u is a homogeneous function of the coordinate x.

One assumes, as it is customary in thermoanemometry, that the pulsating components (or more precisely, their mean-square values) are much smaller than the averaged quantities:

$$u \ll U,\ t \ll T.$$

Under this assumption the multiplier $\mathrm{Re}^{0.5}$ appearing in (1) can be made equal to

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$$\mathrm{Re}^{0.5} = \sqrt{\frac{d}{\nu}}(U+u)^{0.5} \approx \sqrt{\frac{Ud}{\nu}}\left(1+0.5\frac{u}{U}\right). \quad (5)$$

Substituting (5) in (2) and then in (1), two equations are obtained for temperature distribution:

1) for averaged temperatures,

$$\frac{d^2T}{dx^2} - a^2T = -\frac{4Q}{\lambda_2 \pi d^2}; \quad (6)$$

2) for the pulsating temperature component,

$$\frac{d^2t}{dx^2} - a^2t = \varphi(x), \quad (7)$$

where $\varphi(x) = DT(x)u(x)$.

The coefficient $a$ appearing in (6) and (7) can be determined from (2) by using U only.

It has already been pointed out that the temperature at the ends of the wire is constant and equal to the temperature of the surrounding medium. Hence the boundary conditions for Eqs. (6) and (7) are given by

$$T = t = 0 \quad \text{and} \quad x = \begin{cases} l, \\ 0. \end{cases} \quad (8)$$

The solution of Eq. (6) with the boundary conditions (8) is known and is of the form [2]

$$T = D_1\left[1 - \frac{\operatorname{ch} a\left(x - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}}\right]. \quad (9)$$

To find the distribution of the pulsating temperatures one has to solve Eq. (7) together with the boundary conditions (8). This solution is given by

$$t = \frac{\operatorname{sh} ax}{a}\left[\int_0^l \varphi(y)(\operatorname{cth} al \operatorname{sh} ay - \operatorname{ch} ay)\,dy + \int_0^x \varphi(y)\operatorname{ch} ay\,dy\right] - \frac{\operatorname{ch} ax}{a}\int_0^x \varphi(y)\operatorname{sh} ay\,dy. \quad (10)$$

The tension between the ends of the wire is given as the product of the current and its resistance. The relation between the resistance of the unit length of the wire and the temperature in the range of temperatures characteristic for thermoanemometry can be represented by a linear relation

$$\rho = \rho_0[1 + \beta(T + T_1)]. \quad (11)$$

The pulsating part of the resistance is obtained from (11), namely

$$\tilde{\rho} = \rho_0\beta t. \quad (12)$$

The pulsating tension between the wire ends is given in terms of the pulsating temperature by the relation

$$e = I\int_0^l \tilde{\rho}\,dx = I\beta\rho_0\int_0^l t\,dx. \quad (13)$$

Inserting (10) in (13) and by subsequent calculations one obtains the relation between the tension at the ends of the wire and the pulsation velocity,

$$e_m = k\int_0^l \left[\frac{\operatorname{ch} a\left(x - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1\right]^2 u\,dx. \quad (14)$$

If lack of uniformity in the velocity field along the wire can be ignored then (14) can be replaced by

$$e_0 = ku \int_0^l \left[ \frac{\operatorname{ch} a\left(x - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1 \right]^2 dx. \tag{15}$$

The quantity k represents the sensitivity of a unit length of the probe. The evaluation of the integral in (15) yields

$$e_0 = klu \left[ 1 + \frac{1}{2\operatorname{ch}^2\frac{al}{2}} \left( \frac{\operatorname{sh} al}{al} + 1 \right) - \frac{4\operatorname{th}\frac{al}{2}}{al} \right]. \tag{16}$$

With $al \to \infty$ the temperature distribution along the probe becomes uniform and the expression in square brackets is equal to 1. The function

$$g = 1 + \frac{1}{2\operatorname{ch}^2\frac{al}{2}} \left( \frac{\operatorname{sh} al}{al} + 1 \right) - \frac{4\operatorname{th}\frac{al}{2}}{al} \tag{17}$$

gives the sensitivity change due to the cooling at the ends. Its graph is shown in Fig. 1a.

To determine the effect of the ends on the measurement of the variance of the pulsation velocity one has to evaluate the variance integral in (14):

$$\overline{e_m^2} = k^2 \overline{\left\{ \int_0^l \left[ \frac{\operatorname{ch}\left(x - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1 \right]^2 u dx \right\}^2}$$

$$= k^2\overline{u^2} \int_0^l \int_0^l R(x'' - x') \left[ \frac{\operatorname{ch} a\left(x' - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1 \right]^2$$

$$\times \left[ \frac{\operatorname{ch} a\left(x'' - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1 \right]^2 dx'dx''. \tag{18}$$

The bar over a quantity indicates averaging over time.

For further calculations the correlation function R should be given.

For a probe of small dimensions R can be given in the form [2]

$$R(x'' - x') = 1 - \frac{(x'' - x')^2}{\lambda^2}. \tag{19}$$

By substituting (19) in (19) and evaluating the integrals with respect to x' and x'' one obtains

$$\frac{\overline{e_m^2}}{\overline{e_0^2}} = \frac{\overline{u_m^2}}{\overline{u^2}} = 1 - \psi(al)\frac{l^2}{\lambda^2}, \tag{20}$$

where

$$\psi = \left[ \frac{1}{4\operatorname{ch}^2\frac{al}{2}} \left( \frac{a^2l^2 + 2}{a^3l^3} \operatorname{sh} al - \frac{2\operatorname{ch} al}{a^2l^2} + \frac{1}{3} \right) - 2\left( \frac{l^2a^2 + 8}{4a^3l^3} \operatorname{th}\frac{al}{2} \right.\right.$$

$$\left.\left. - \frac{1}{a^2l^2} \right) + \frac{1}{6} \right] \left[ 1 + \frac{1}{2\operatorname{ch}^2\frac{al}{2}} \left( \frac{\operatorname{sh} al}{al} + 1 \right) - \frac{4\operatorname{th}\frac{al}{2}}{al} \right]^{-1}.$$

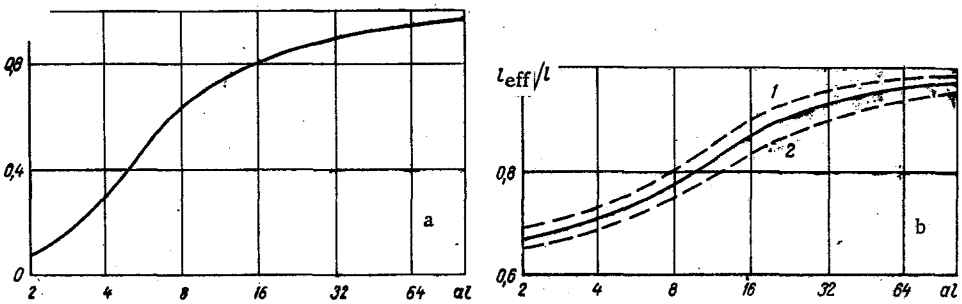


Fig. 1. The sensitivity of the wire probe (a) and of the effective length of the wire (b) vs dimensionless parameter *al*. In a), the ordinate axis represents g.

The relation (20) is similar to the one obtained previously in [1] for a probe with uniform temperature distribution. Both relations can be rewritten in one formula,

$$\frac{\overline{u_m^2}}{\overline{u^2}} = 1 - \frac{1}{6}\frac{l_{\text{eff}}^2}{\lambda^2}. \tag{21}$$

For a uniform distribution of temperatures (that is, for $al \to \infty$) one has $l_{\text{eff}} = l$ and for nonuniform $l_{\text{eff}}^2 = 6\psi l^2$.

For $l \gg L$ the correlation function changes much more rapidly than the deterministic functions appearing under the integral sign in the expression (18). The correlation function can, therefore, be represented by

$$R = \delta(x'' - x')L, \text{ where } L = \int_0^\infty R(r)\,dr. \tag{22}$$

Substituting (22) into (18) one obtains

$$\int_0^l\int_0^l R(x'' - x')G(x'')G(x')\,dx''dx'$$

$$= L\left\{\int_0^l G(x')\left[\int_0^{x'} \delta(x'' - x')G(x'')\,dx'' + \int_{x'}^l \delta(x'' - x')G(x'')\,dx''\right]dx'\right\}$$

$$= 2L\int_0^l G^2(x)\,dx, \tag{23}$$

where

$$G = \left[\frac{\operatorname{ch} a\left(x - \frac{l}{2}\right)}{\operatorname{ch}\frac{al}{2}} - 1\right]^2.$$

Hence

$$\frac{\overline{e_m^2}}{\overline{e_0^2}} = \frac{\overline{u_m^2}}{\overline{u^2}} = \frac{2L\int_0^l G^2dx}{\left\{\int_0^l Gdx\right\}^2} = \frac{2L}{l_{\text{eff}}}, \tag{24}$$

where

$$l_{\text{eff}} = \frac{\left\{\int_0^l Gdx\right\}^2}{\int_0^l G^2dx}.$$

For $al \to \infty$ one obtains $l_{eff} = l$ and the relation (24) is transformed into the one previously obtained in [1] for a uniform temperature distribution. The graphs of $l_{eff}/l$ vs $al$ for a long probe (Curve 1) or a short one (Curve 2) are shown in Fig. 1b by dashed lines. Of course, one has

$$\left(\frac{l_{eff}}{l}\right)_{l\to 0} \neq \left(\frac{l_{eff}}{l}\right)_{l\to\infty}.$$

The difference between them, however, is not large; therefore, both graphs can be replaced by a single one obtained as their arithmetic average (Fig. 1b — continuous line). The deviation of the averaged line from the original ones does not exceed 3% in magnitude.

One was thus able to reduce the effect of nonuniformity in the temperature distribution along the probe by introducing the concept of effective length. The cooling effect at the ends of the probe by brackets shortens the effective probe length and increases its resolving capacity.

It follows from the obtained results that the magnitude of this effect increases with the reduction of the heat exchange intensity or of the ratio of probe length to its thickness. However, in practice the use of such a probe with a higher resolving capacity is somewhat involved. A comparatively slight rise in the resolving capacity involves lowering of the probe sensitivity as one can see by comparing Fig. 1a and b.

For example, for a wolfram wire with $d = 20 \cdot 10^{-6}$ m, $l = 10^{-3}$ m with air velocity 5 m/sec one has $al = 2$; $l_{eff}/l = 0.65$; $g = 0.05$. It is doubtful whether the use of probes with a lower value of $al$ is worth while in view of low sensitivity.

## NOTATION

| | |
|---|---|
| $\theta$ | is the temperature difference between the wire and the surrounding medium; |
| x | is the coordinate along the wire measured from one of its ends; |
| Nu | is the Nusselt number; |
| $\alpha$ | is the heat exchange coefficient of the wire; |
| $\lambda_1$, $\lambda_2$ | are the coefficients of heat conduction for the medium and the wire; |
| d | is the wire diameter; |
| Q | is the heat generation in unit of time on a unit of wire length; |
| I | is the intensity of the current flowing through the wire; |
| $\rho$ | is the resistance of a unit of wire length; |
| Pr | is the Prandtl number; |
| Re | is the Reynolds number; |
| U, u | are the average and puslation flow velocities; |
| T, t | are the average and pulsation components of the probe overheating relative to the temperature of the medium; |
| D, $D_1$ | are the constant coefficients; |
| $T_1$ | is the flow temperature; |
| $\tilde{\rho}$ | is the pulsation component of resistance of a wire unit length; |
| $\rho_0$ | is the resistance of wire unit length at 0°C; |
| e | is the pulsating voltage between the ends of the wire; |
| $e_m$ | is the pulsating voltage on probe subject to cooling effect of brackets; |
| $e_0$ | is the voltage pulsation on probe with uniform velocity distribution; |
| $\bar{u}^2$ | is the variance of velocity pulsation; |
| $\bar{e}^2$ | is the variance of pulsating voltage; |
| x', x" | are the coordinates of any points on the probe; |
| R | is the correlation function; |
| $\lambda$ | is the turbulence microscale; |
| $l_{eff}$ | is the effective length of probe when the turbulent velocity pulsation is measured; |
| L | is the integral scale of turbulence; |
| $\delta$ | is the delta function. |

## LITERATURE CITED


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2. I. O. Khintse, Turbulence [in Russian], Fizmatgiz, Moscow (1963).